Finite-size effects and the search for the Critical Endpoint in HIC's

Letícia F. Palhares
Eduardo S. Fraga
Takeshi Kodama

Instituto de Física, UFRJ

Instituto de Física
Universidade Federal do Rio de Janeiro

CNPq
Conselho Nacional de Desenvolvimento Científico e Tecnológico

FAPERJ

Searching the Critical Endpoint in HIC's

Some of the most popular *signatures* of the **Critical Endpoint (CEP; 2nd order transition; $\xi \to \infty$)** are based on the expected *divergent* behavior of the correlation functions of the quasi-particle σ, related to the order parameter of the chiral transition $\langle \bar{\psi}\psi \rangle$:

[Stephanov, Rajagopal, Shuryak (98,99); Berdnikov, Rajagopal (2000);Stephanov(2009)]

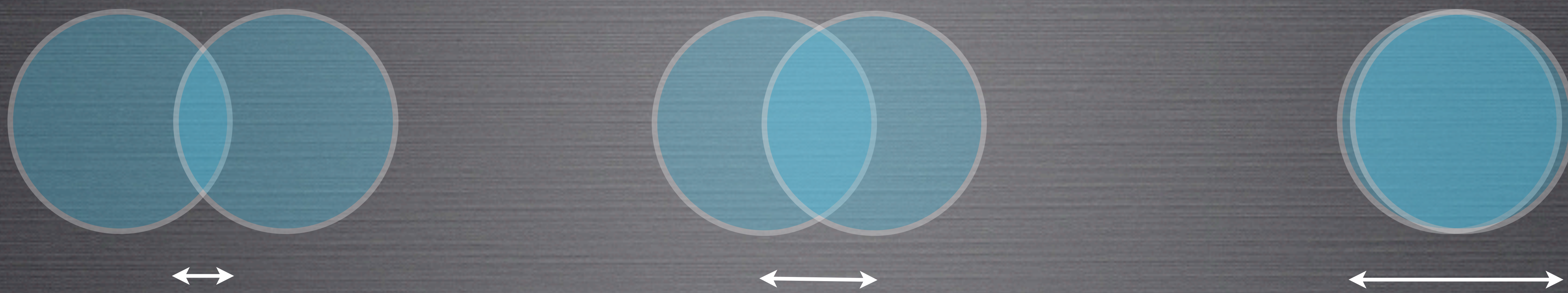
$$\boxed{\langle \sigma^n \rangle \sim \xi^{p_n}}$$

As is well-known, in any real system, the correlation length ξ is always finite, and a *nonmonotonic* behavior is expected instead.

This feature should be translated into the final observable spectra in HIC via mesonic decays of the sigma field into other particles, especially *soft* pions (created as soon as the medium-dependent sigma reaches the mass threshold).

The system created in HIC's is FINITE, and its size is CENTRALITY-DEPENDENT:

$L(N_{\mathrm{part}})$

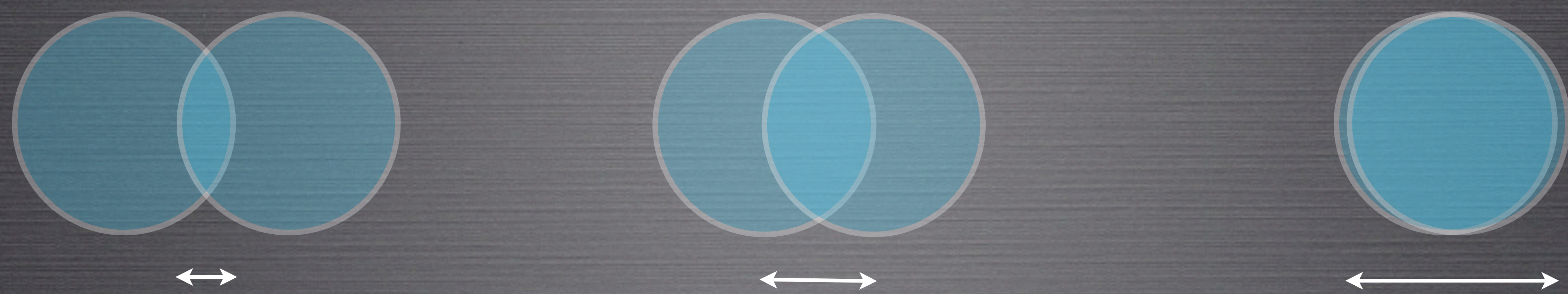


$L \sim 2$ fm $\qquad$ $L < 10$ fm $\qquad$ $L \sim 10 - 15$ fm

How can this feature influence/affect possible signatures of the CEP?

The system created in HIC's is FINITE, and its size is CENTRALITY-DEPENDENT:

$L(N_{\text{part}})$

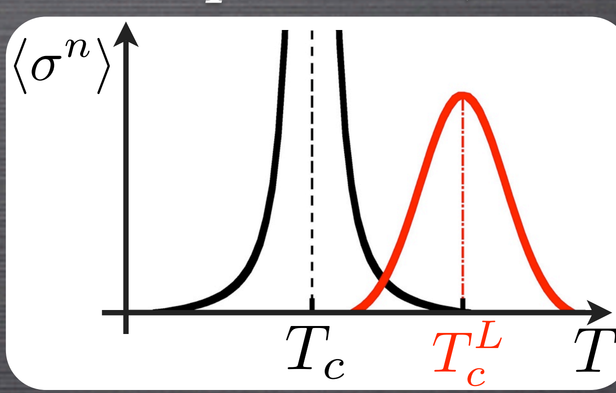


$L \sim 2$ fm $\qquad$ $L < 10$ fm $\qquad$ $L \sim 10 - 15$ fm

How can this feature influence/affect possible signatures of the CEP?

(1) Most signatures will probe *pseudocritical quantities*, with **smoothened divergences** and **shifted peaks**:

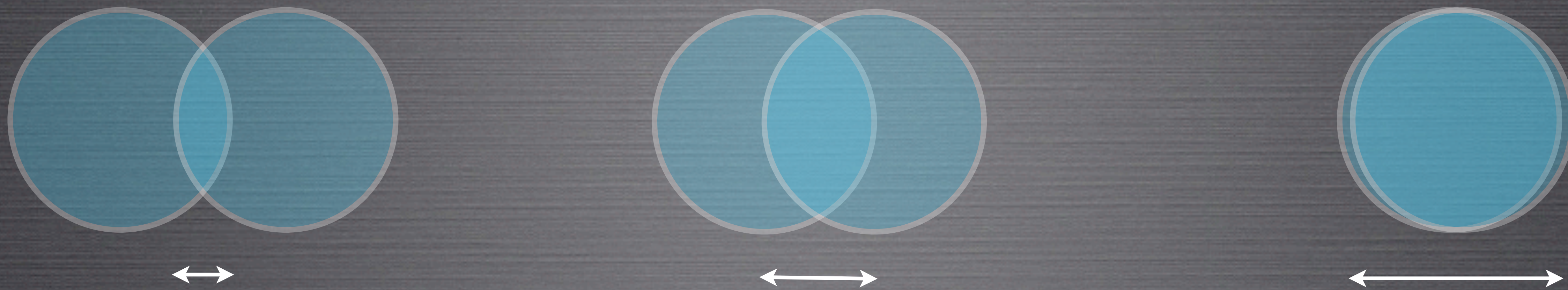
$$\langle \sigma^n \rangle_L \sim \xi^{p_n} \; f_n(\xi/L) \quad \Rightarrow$$



Investigate the importance of the shifts within a chiral model for typical HIC size scales and the consequences for the related signatures.

The system created in HIC's is FINITE, and its size is CENTRALITY-DEPENDENT:

$L(N_{\text{part}})$

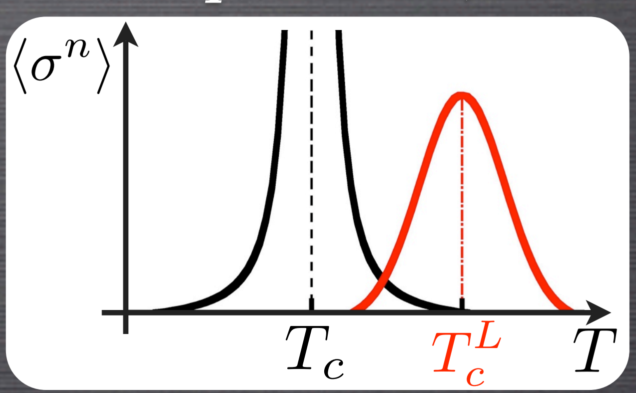


$L \sim 2$ fm $L < 10$ fm $L \sim 10 - 15$ fm

How can this feature influence/affect possible signatures of the CEP?

(1) Most signatures will probe *pseudocritical quantities*, with **smoothened divergences** and **shifted peaks**:

$$\langle \sigma^n \rangle_L \sim \xi^{p_n} \, f_n(\xi/L) \Rightarrow$$



Investigate the importance of the shifts within a chiral model for typical HIC size scales and the consequences for the related signatures.

(2) HIC data as an **ensemble of systems of different sizes**.

Finite-size analysis as a tool to obtain info about the true phase transition, in the thermodynamic limit: location of the CEP and its universality class.

the Linear σ Model with quarks at finite volume

$$\mathcal{L} \;=\; \overline{\psi}_f \left[i\gamma^\mu \partial_\mu + \mu\gamma^0 - g\sigma \right] \psi_f + \frac{1}{2}\partial^\mu\sigma\partial_\mu\sigma - \left[\frac{\lambda}{4}(\sigma^2 - v^2)^2 - h\sigma \right]$$

Models the Chiral Properties of QCD: spontaneous and small explicit breaking

Parameters fixed to reproduce observed properties of the QCD vacuum

Pions dropped for simplicity, since they do not affect much the phase structure

[Scavenius *et al* (2001)]

the Linear σ Model with quarks at finite volume

$$\mathcal{L} = \overline{\psi}_f \left[i\gamma^\mu \partial_\mu + \mu\gamma^0 - g\sigma \right] \psi_f + \frac{1}{2}\partial^\mu \sigma \partial_\mu \sigma - \left[\frac{\lambda}{4}(\sigma^2 - v^2)^2 - h\sigma \right]$$

Models the Chiral Properties of QCD: spontaneous and small explicit breaking

Parameters fixed to reproduce observed properties of the QCD vacuum

Pions dropped for simplicity, since they do not affect much the phase structure

[Scavenius *et al* (2001)]

Finite Volume: $V \int \frac{d^3\vec{k}}{(2\pi)^3} f(\vec{k}) \mapsto \sum_n f\left(\vec{k}(n)\right)$ Boundary Conds.: $\vec{k}(n)$

THE LINEAR σ MODEL WITH QUARKS AT FINITE VOLUME

$$\mathcal{L} = \overline{\psi}_f \left[i\gamma^\mu \partial_\mu + \mu\gamma^0 - g\sigma \right] \psi_f + \frac{1}{2}\partial^\mu \sigma \partial_\mu \sigma - \left[\frac{\lambda}{4}(\sigma^2 - v^2)^2 - h\sigma \right]$$

Models the Chiral Properties of QCD: spontaneous and small explicit breaking

Parameters fixed to reproduce observed properties of the QCD vacuum

Pions dropped for simplicity, since they do not affect much the phase structure

[Scavenius *et al* (2001)]

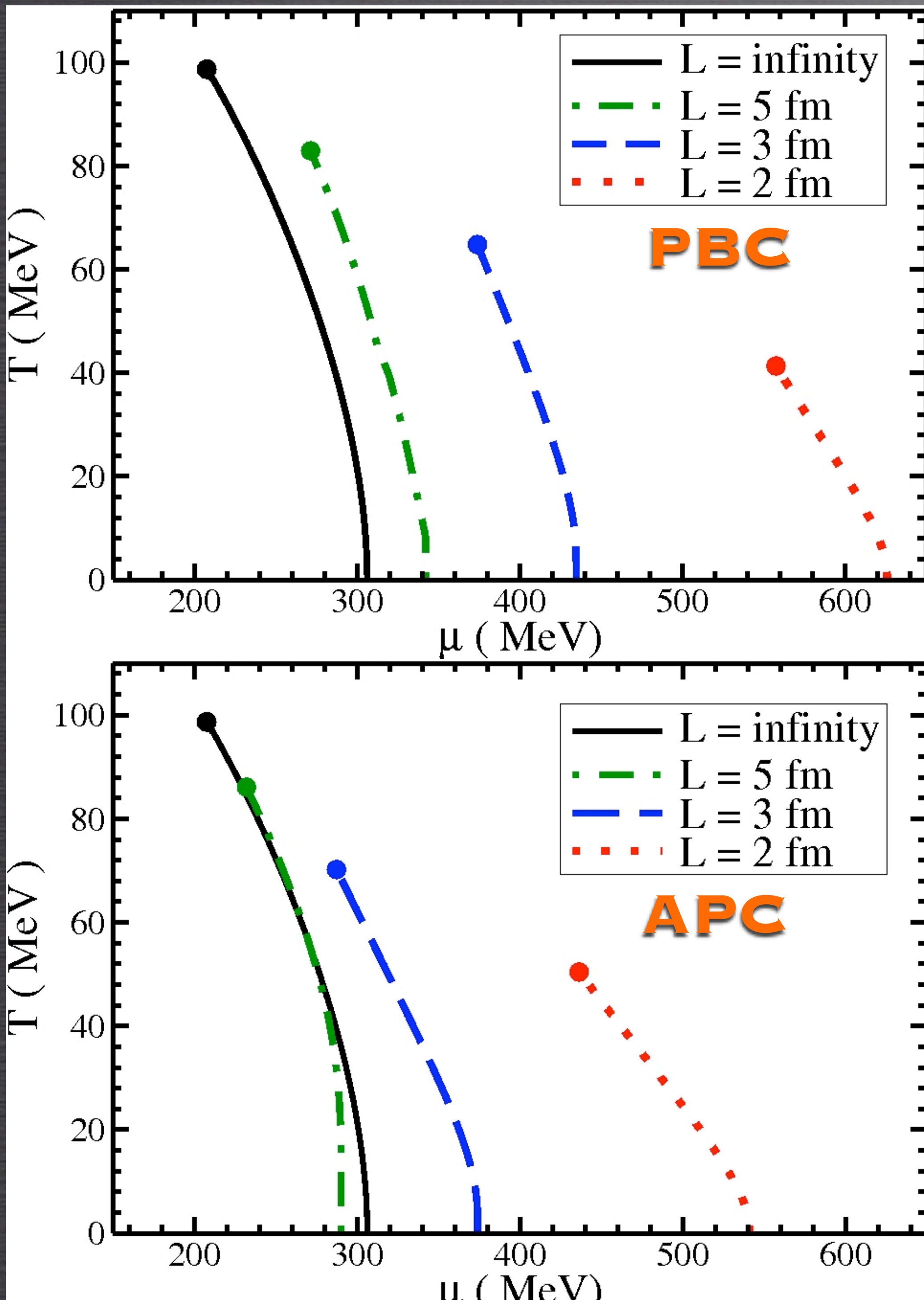
FINITE VOLUME: $V \int \frac{d^3\vec{k}}{(2\pi)^3} f(\vec{k}) \mapsto \sum_n f\left(\vec{k}(n)\right)$ BOUNDARY CONDS.: $\vec{k}(n)$

MAIN GOAL HERE: ESTIMATE AMPLITUDE OF SHIFTS IN THE (PSEUDOCRITICAL) PHASE DIAGRAM OF THE CHIRAL TRANSITION.

- **Mean-field calculation (to compare with standard mean-field results in the thermodynamic limit);** neglecting fluctuations, concentrating on bulk effects
- **Illustrate boundary-condition dependence by analyzing *Periodic BC (zero spatial mode)* and *Antiperiodic BC***

The Pseudocritical Phase Diagram of the chiral transition

[LFP, Fraga, Kodama (2009)]

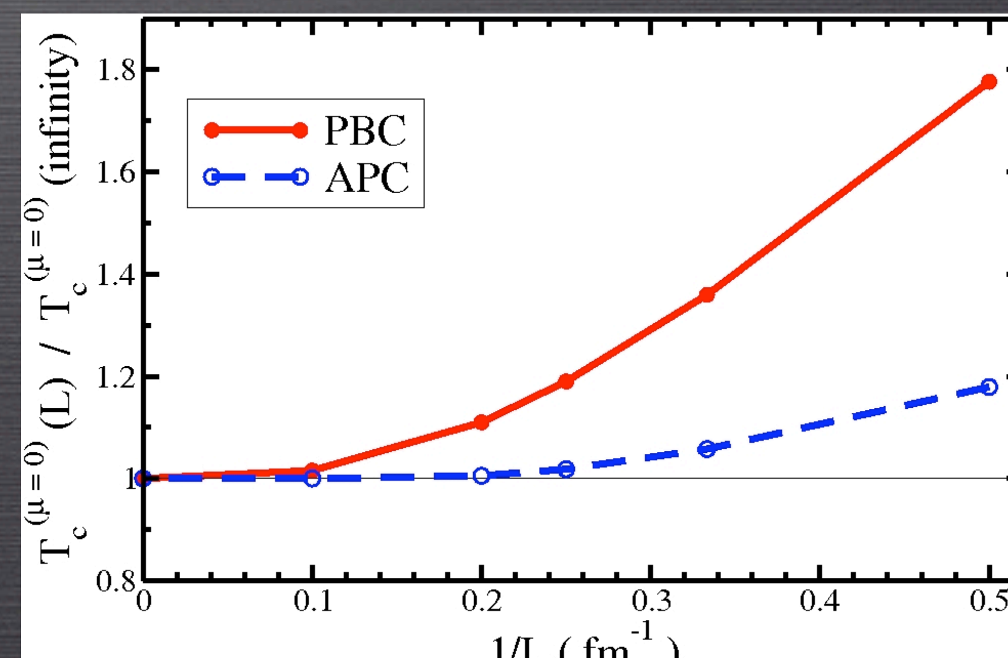


- large effects for size ranges relevant to current HIC's

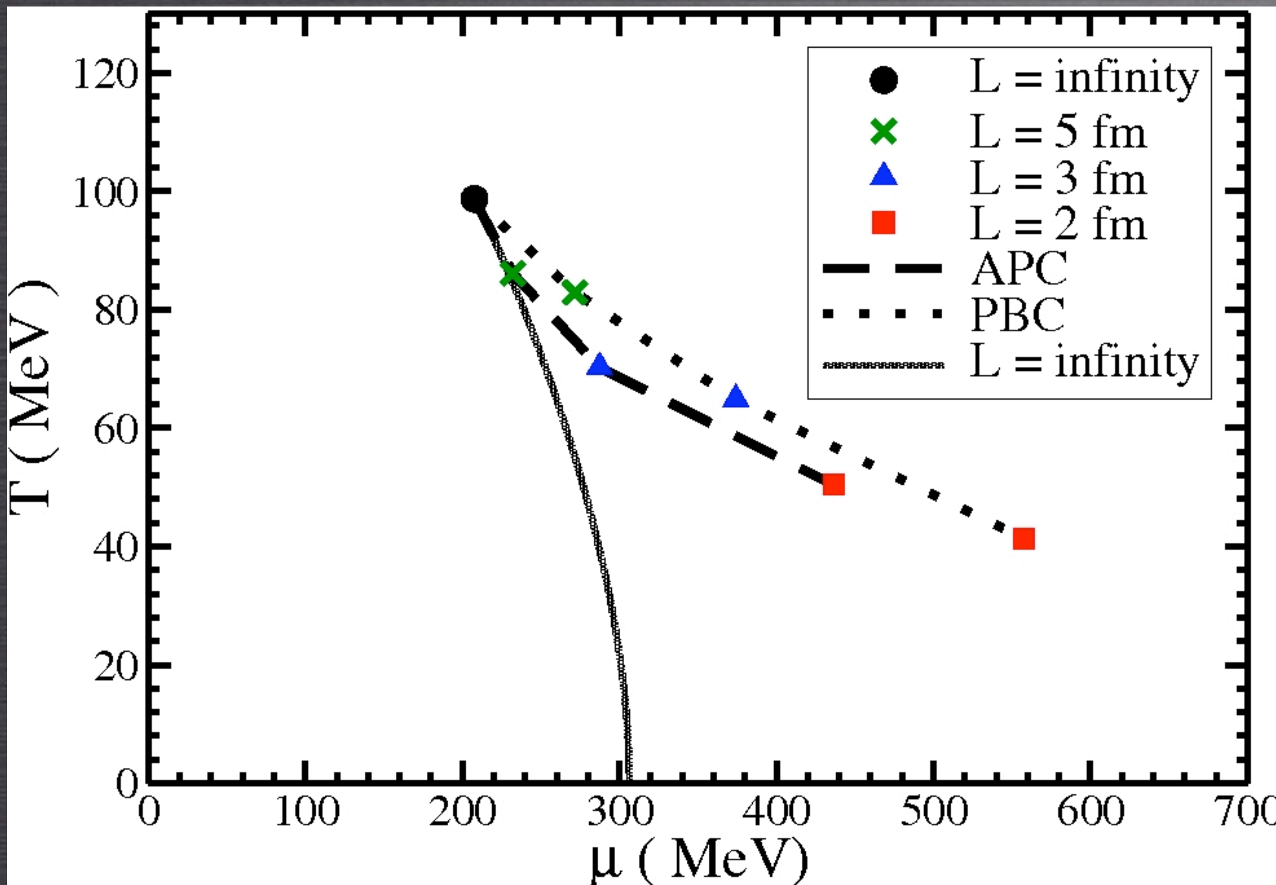
- transition line and CEP significantly shifted to large μ ⇒ outside experimental reach?

- Strong boundary-condition dependence

Crossover Temperature at zero chemical potential



THE (PSEUDO)CEP: VOLUME AND BC DEPENDENCE

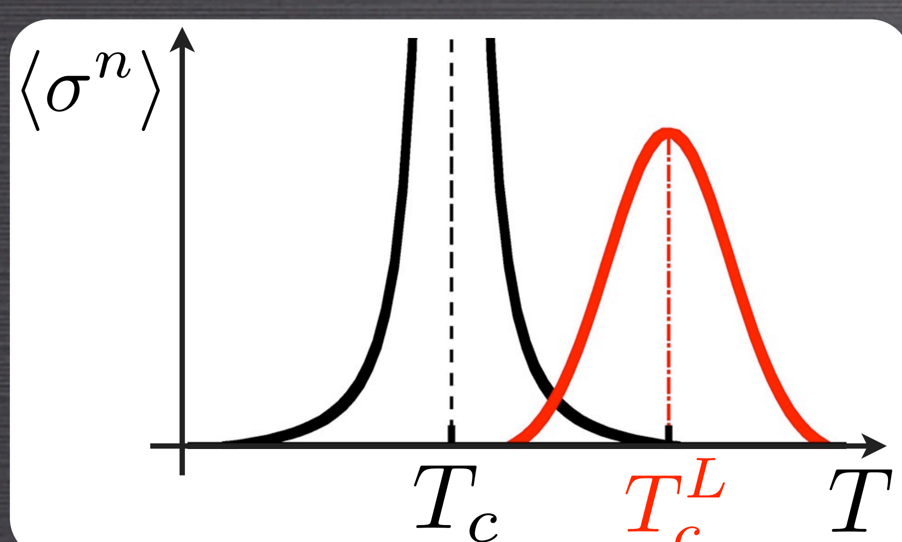


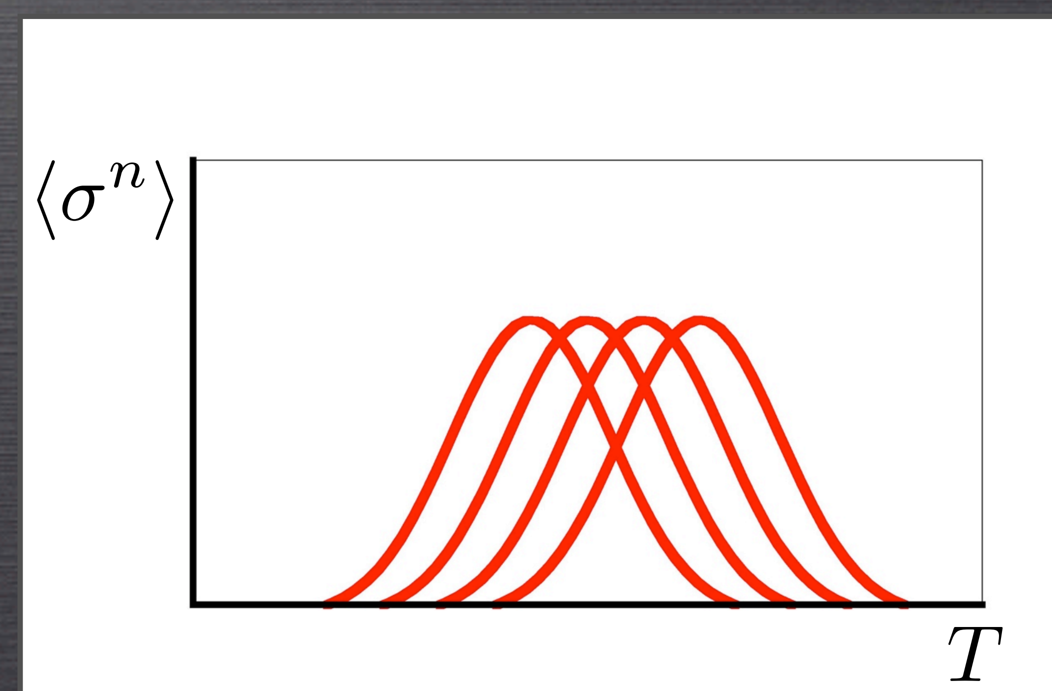
Decreasing L: CEP shifts to lower T_{CEP} and higher μ_{CEP};

μ_{CEP} has stronger BC dependence.

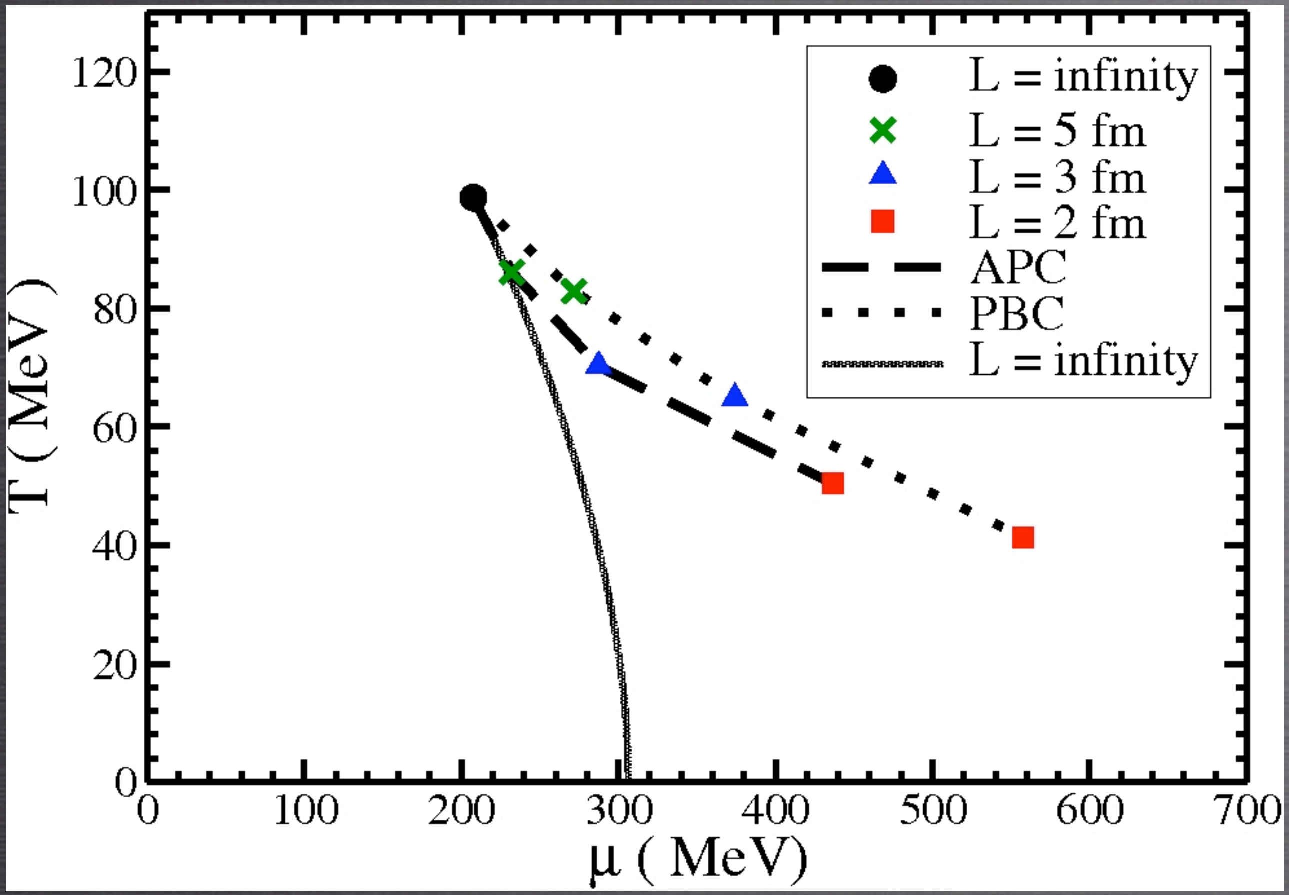
Significant corrections for size scales probed in HIC's

For signatures related to the nonmonotonic behaviour of the order parameter correlations averaged within a centrality window:





THE (PSEUDO)CEP: VOLUME AND BC DEPENDENCE

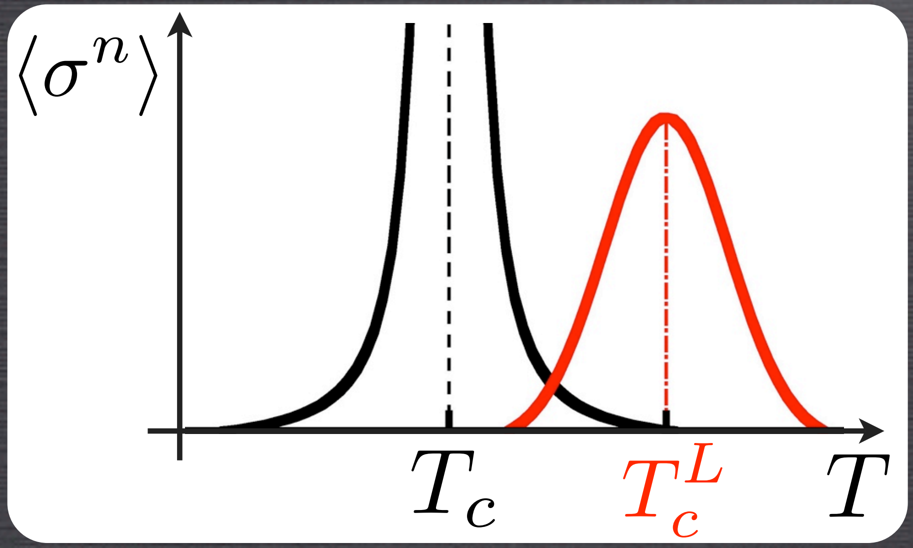


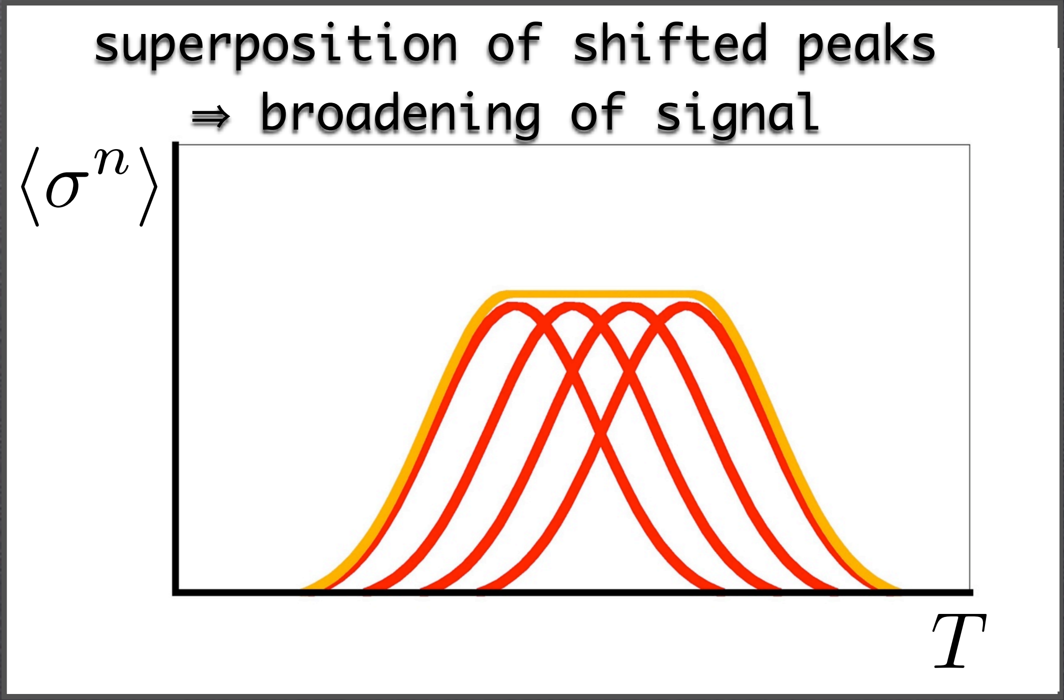
Decreasing L: **CEP shifts to lower** T_{CEP} **and higher** μ_{CEP};

μ_{CEP} **has stronger BC dependence.**

Significant corrections for size scales probed in HIC's

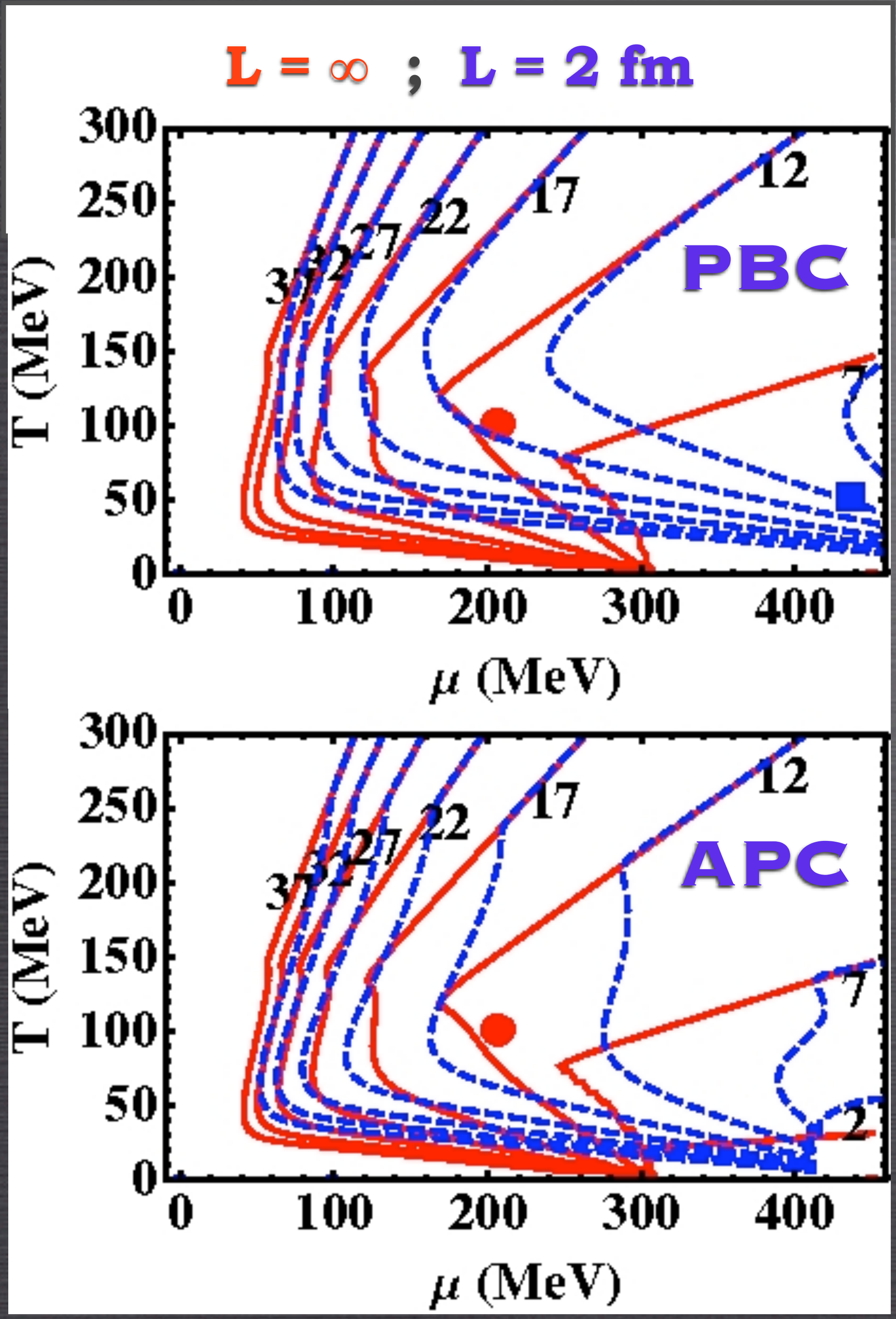
For signatures related to the nonmonotonic behaviour of the order parameter correlations averaged within a centrality window:





Isentropic Trajectories at finite volume

[LFP, Fraga, Kodama (2009)]



- large effects in the critical region.

- analysis based on (nearly) isentropic hydro evolutions could be greatly affected.

FSS as a tool for searching the CEP...

CEP $\Rightarrow$ 2nd order phase transition $\longrightarrow$ $\begin{cases} \text{divergent correlation length} \\ \text{scale invariance on the criticality} \end{cases}$

FSS as a tool for searching the CEP...

CEP ⇒ 2^{nd} order phase transition → divergent correlation length; scale invariance on the criticality

These features imply the existence of Finite-size Scaling for finite systems in the vicinity of the CEP (rigorous proof through RG analysis):

$$\boxed{X(t,L) \quad = \quad L^{\gamma_x/\nu} f_x(tL^{1/\nu})}$$

$t = (T - T_c)/T_c$ **(distance to the *genuine* CEP)**

X ⇒ (any) correlation function of the order parameter

ν ⇒ universal critical exponent (div. of corr. length)

FSS as a tool for searching the CEP...

CEP ⇒ 2ND order phase transition →
- divergent correlation length
- scale invariance on the criticality

These features imply the existence of Finite-size Scaling for finite systems in the vicinity of the CEP (rigorous proof through RG analysis):

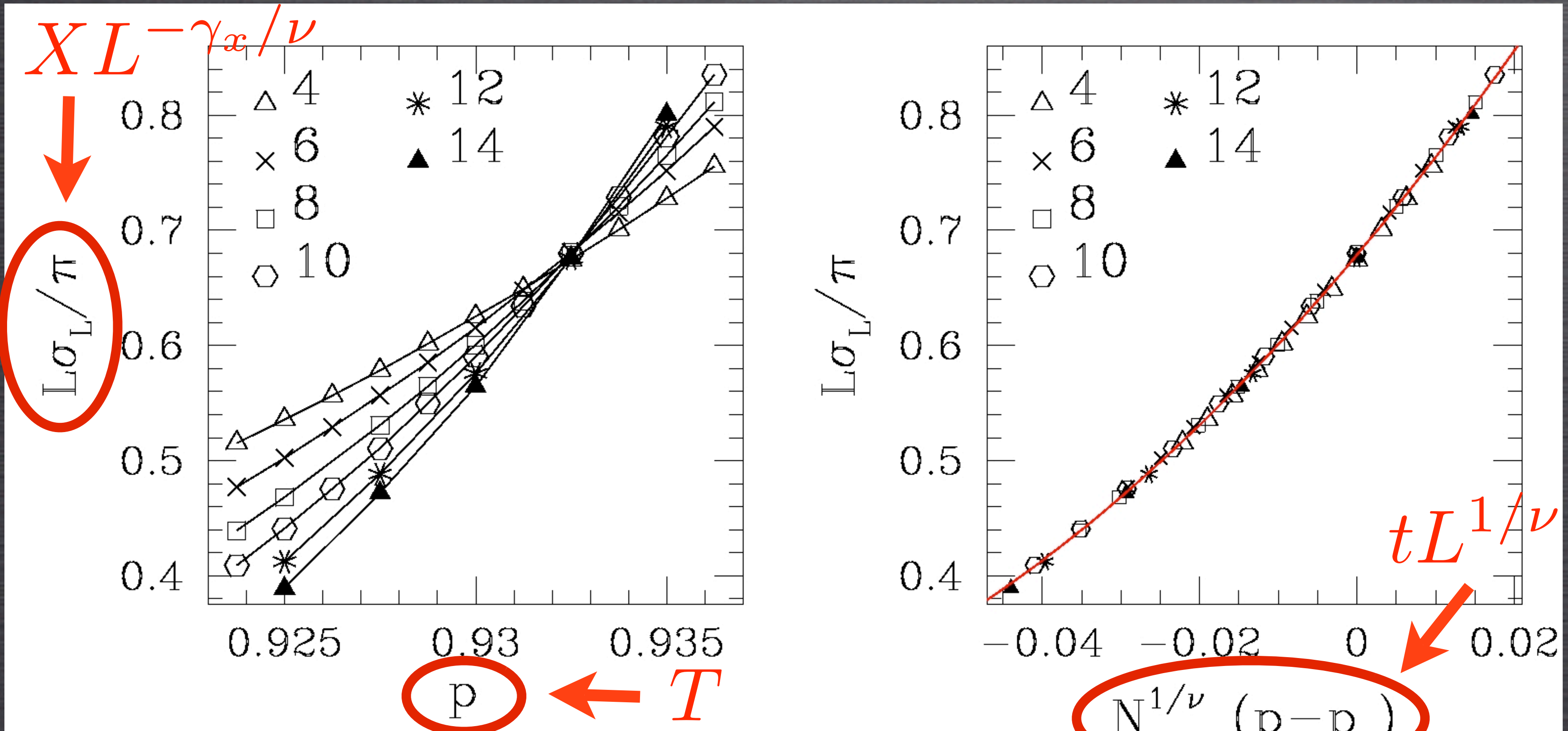
$$\boxed{X(t,L) \quad = \quad L^{\gamma_x/\nu} f_x(tL^{1/\nu})}$$

$t=(T-T_c)/T_c$ **(distance to the *genuine* CEP)**

X ⇒ (any) correlation function of the order parameter

ν ⇒ universal critical exponent (div. of corr. length)

Scaling plots in Condensed Matter: Spin Glass transitions in disordered Ising systems



[de Queiroz, PRB 79, 174408 (2009)]

Tool for determining T_c and critical exponents (universality class)

NOTE: signature present even in observables that show no nonmonotonic behavior

FSS as a tool for searching the CEP...

... in HIC's

- **HIC data as experimental realization of an *ensemble of systems of different sizes/centralities* ⇒ we can investigate the size dependence of the observables**

- **Since FSS ⇔ CEP, then identifying FSS in the centrality dependence of HIC**

FSS: $X(t,L) = L^{\gamma_x/\nu} f_x(tL^{1/\nu})$ $\longrightarrow$ $\left\{\begin{array}{l}\text{locate CEP} \\ \text{determine its critical exponents (universality class of QCD)}\end{array}\right.$

FSS as a tool for searching the CEP...
... in HIC's

- **HIC data as experimental realization of an** ***ensemble of systems of different sizes/centralities*** ⇒ **we can investigate the size dependence of the observables**

- **Since FSS ⇔ CEP, then identifying FSS in the centrality dependence of HIC**

FSS: $X(t,L) = L^{\gamma_x/\nu} f_x(tL^{1/\nu})$ ⟶ $\begin{cases} \text{locate CEP} \\ \text{determine its critical exponents (universality class of QCD)} \end{cases}$

How can we look for FSS in HIC data? **Verifying if data is compatible with the FSS relation:**

FSS relation in the case of HIC

(we need observables related to t, L, etc ***only up to normalization consts.***)

- distance t to the CEP: $t \mapsto (\sqrt{s} - \sqrt{s_c})/\sqrt{s_c}$
- size L of the system: $L \mapsto N_{\rm part}^{1/2}$
- Correlation function X of the order parameter: $\longmapsto$ correlations of soft pion fluctuations, e.g.

$$\boxed{X\ N_{\rm part}^{-\gamma_x/2\nu} = f_x\left(y_{\rm scl}\right)}$$

scaling variable: $y_{\rm scl} = \dfrac{\sqrt{s}-\sqrt{s_c}}{\sqrt{s_c}} N_{\rm part}^{1/2\nu}$

Scaling plots in HIC's

FSS analysis can be implemented ***ON*** the CEP ($t = 0$) **[Lizhu, Chen, Yuanfang (2009); *see talk by Y. Wu*]** ***OR VIA***

FULL SCALING PLOTS **[LFP, Fraga, Kodama (2009)]**

- Necessary ***and*** sufficient condition for FSS (and thus for the presence of the CEP)
- Should be valid in a **larger vicinity** of the CEP and could be tested even if there is data only above ***or*** below the CEP

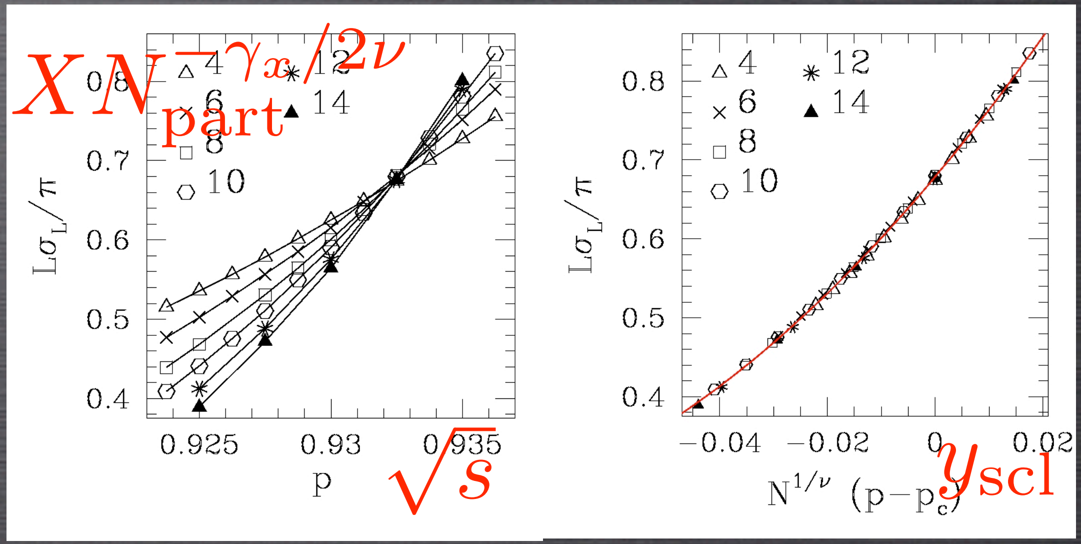
Scaling plots in HIC's

FSS analysis can be implemented ***ON*** the CEP (t = 0) **[Lizhu, Chen, Yuanfang (2009); *see talk by Y. Wu*]** ***OR VIA***

FULL SCALING PLOTS **[LFP, Fraga, Kodama (2009)]**

- Necessary ***and*** sufficient condition for FSS (and thus for the presence of the CEP)
- Should be valid in a **larger vicinity** of the CEP and could be tested even if there is data only above ***or*** below the CEP

ILLUSTRATION:



PROCEDURE: **Search for γ_x, ν, $\sqrt{s}_c$ which *collapse* data from different centralities in the associated scaling plot ($XN_{\rm part}^{-\gamma_x/2\nu} \times y_{\rm scl}$)**

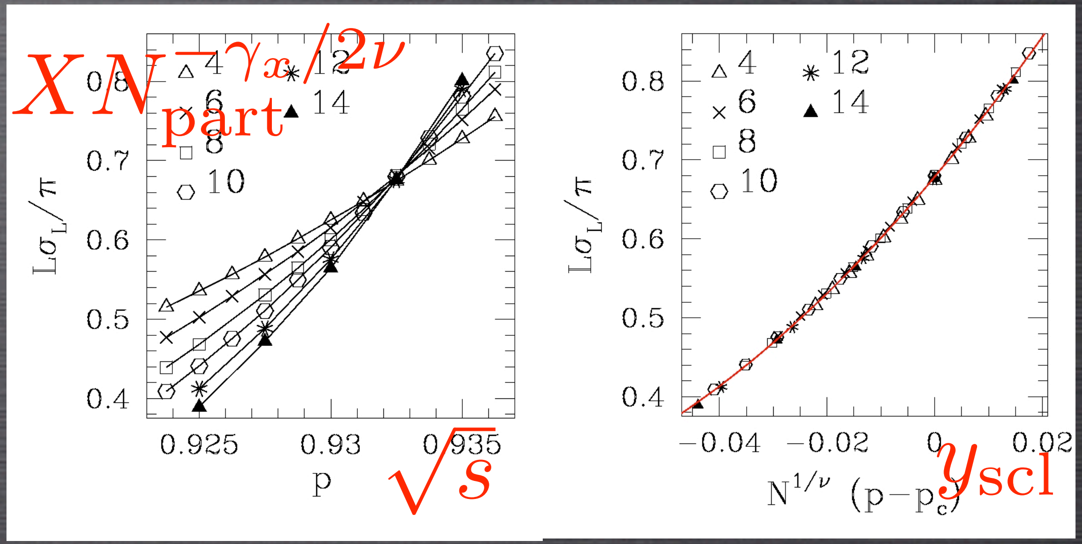
Scaling plots in HIC's

FSS analysis can be implemented ***ON*** the CEP ($t = 0$) **[Lizhu, Chen, Yuanfang (2009); *see talk by Y. Wu*]** ***OR VIA***

FULL SCALING PLOTS **[LFP, Fraga, Kodama (2009)]**

- Necessary ***and*** sufficient condition for FSS (and thus for the presence of the CEP)
- Should be valid in a **larger vicinity** of the CEP and could be tested even if there is data only above ***or*** below the CEP

Illustration:



PROCEDURE: **Search for γ_x, ν, $\sqrt{s}_c$ which *collapse* data from different centralities in the associated scaling plot ($XN_{\rm part}^{-\gamma_x/2\nu} \times y_{\rm scl}$)**

METHODS FOR DATA ANALYSIS:

Fits (standard method in statistical mechanics)

χ^2 method: minimize the difference between data points associated with the same value of the scaling variable:

$$\chi^2\left(\nu, \sqrt{s}_c; y_0 = N_{\rm part,0}^{\nu/2}\frac{\sqrt{s}_0 - \sqrt{s}_c}{\sqrt{s}_c}\right) = \sum_{y_{\rm scl}(\sqrt{s}, N_{\rm part}) = y_0} \left(\frac{X(\sqrt{s}, N_{\rm part}, \nu) N_{\rm part}^{-\gamma_x/2\nu}}{X(\sqrt{s}_0, N_{\rm part,0}, \nu) N_{\rm part,0}^{-\gamma_x/2\nu}} - 1\right)^2$$

Conclusions

Finite-size effects can play a crucial role in the search for the CEP in HIC's in BES-RHIC and FAIR-GSI.

(1) **Nonmonotonic (or sign-change) signatures will probe *pseudocritical* observables,** that are smoothened and shifted from the genuine criticality in the thermodynamic limit by corrections dependent on size/centrality and boundary conditions.

We show within the LσM that:

- **corrections can be large for the size scales involved in current HIC**
- **the (pseudo)critical line is shrinked and shifted to the higher μ regime, as the size decreases.**
- **isentropic trajectories change significantly around the critical region**

Due to the size/centrality dependent shifts of the pseudocritical peaks, averages over not sufficiently small centrality windows could generate a broader nonmonotonic signature, contributing to wash it out in the thermal background.

⇒ Data analysis in small centrality bins

Conclusions(II)

(2) HIC data can be seen as an ensemble of systems of different sizes

⇒ ***Finite-size scaling*** can be a useful tool in the search for the CEP in HIC in BES-RHIC and FAIR-GSI. Its presence represents an independent and complementary signature of the 2nd order CEP and can give info about the phase diagram in the thermodynamic limit (including the universality class).

We propose the application of full scaling plots in the search for FSS in HIC data, and discuss a χ^2-method as one possible systematic tool for data analysis.

As well as most of the other CEP signatures, the FSS signature relies on the fact that we can connect correlations in final particle spectra to correlation functions of the order parameter of the transition.

Possible tests of the FSS signature:
MC simulations of the evolution of correlations in the thermal background;
FSS analysis in low-energy nuclear CEP data.